Wage-Employment-Driven Business Cycle Model

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Abstract

This paper presents a wage-employment-driven, endogenous, no-trend, business cycle dynamic model (BuCDyn). The math model is developed to fill a void for a working simple model. Computed results compare favorably with U.S. economic data around 2007. Qualitative features of the computed business cycle include a period of 5 years, an expansion of 3 years, and recession of two years. We introduce variable wage and labor time constants, depending on position within the business cycle. Actual and computed leading, coincident, and lagging economic time series are illustrated and compared. Our results indicate that actual business cycles are most likely never in equilibrium with unsteadiness always present. This feature may explain why various qualitatively similar business cycles are never the same.

Keywords business cycle, computational business cycle dynamics, endogenous, exogenous, expansion, macro-dynamics, math model, ordinary differential equation, recession, U. S. economy

1 Introduction and motivation

Understanding the business cycle is important for policy makers, central bankers, business decision makers, and everyday citizens. Clearly, expansion and recession phases of the overall economy have significant implications for income, expense, and opportunity. The last U. S. recession, from December 2007 to June 2009 (NBER, 2011) was difficult for many business owners, employees, and bankers because available credit fell as unemployment and mortgage defaults rose. The high cost of unemployment, lost production, lost confidence, and increase of business uncertainty is not quantifiable and cannot be quickly regained. The Federal Reserve is charged with managing the business cycle through mandates to provide both employment and price stability. Yet, comprehensive Federal Reserve business cycle models and or policy may be incomplete as the last recession was the most severe since the early 1930s. The motivation for developing our math model of business cycle dynamics is that a working simple model could not be identified from an extensive literature search.

Business cycle literature is briefly reviewed in Section 2. In Section 3, we develop our math model of the business cycle dynamics. In Section 4 we discuss general observations about the U.S. economy including Gross Domestic Production (GDP) distribution. Also, national income accounting is briefly reviewed as it pertains to the present work. We compute business cycle variables around 2007 and compare with actual values in Section 5. We summarize the results of this paper in Section 6.

2 Literature review

Three theories that try to explain business cycles and growth include: neo-classical or exogenous, real business cycle (RBC), and endogenous business cycle (EBC). These theories are briefly reviewed below.

The first aggregate production function is developed by Cobb and Douglas (1928). They curve fit U.S. manufacturing sector data from 1899 to 1922 and determined that GDP (Y) = b $L^{3/4} K^{1/4}$ (1928:155), where b is independent of labor (L) and capital stock (K). This formula illustrates how labor and capita stock characterize production level, GDP or Y. Samuelson (1979) suggested that the exponents in the aggregate production function are fundamental and come from an income distribution result. Felipe and Adams (2005: 433) showed that the Cobb-Douglas production function exponents can be derived from an income accounting identity where real value added equals total wages and total profits. Neo-classical growth models usually converge to a steady state growth rate consistent with external (to the model) labor force and technology growth.

The Harrod (1939) - Domar (1946) model was an early neo-classical exogenous (outside the model) growth model. Their model determines that production growth rate equals savings rate times the marginal product of capital (Y/K) minus depreciation rate. In this model the ratios of capital/output (K/Y) and the capital/labor (K/L) are fixed. Solow (1956) and Swan (1956) both independently extended the Harrod - Dommar model by adding labor as a factor of production with a Cobb-Douglas function. Solow introduces a time or technology dependent scale factor, A(t), in a Cobb-Douglas function (1956:85), and employs variable K/Y and K/L ratios. Solow (1957) computes that A(t) varies from a reference value of 1.0 in 1909 to 1.81 in 1949. He finds the increase in A(t) due to an increase in the level of technology that is exogenous to the model. For the same data, Solow (1957: 318) crafted a mathematical relationship, employing the Cobb-Douglas function, between dependent variable Y/(LA) and independent variable K/L. He fit a log equation to the data. This equation can be reformulated into a Cobb-Douglas function with a 0.647 exponent on L and a 0.353 exponent on K. When the exponents in a Cobb-Douglas function add to one, production, Y, increases by 100% when labor, L, and capital stock, K both increase by 100%. This feature is called constant-return-to-scale.

Real business cycle (RBC) models have business cycle drivers of external or exogenous "real" (not monetary) shocks and assume the economic system is otherwise stable, see Kydland and Prescott (1982). They combine growth and business cycle theory. Exogenous stochastic components can be technology shocks. These models typically seek to replicate statistical features of the U.S. business cycle. A common approach is to detrend time series with a filter like Hodrick and Prescott (1980) prior to computing statistics. Employing U.S. data from 1950-1 to 1979-II, Kydland and Prescott find that standard deviations in % about their mean are: output 1.8%, investment 5.1%, total consumption 1.3%, and total hours worked 2.0%. Their correlation coefficient, R^2, vs. real-output are: total consumption 0.74, fixed investment 0.71, and total hours worked 0.85; see Kydland and Prescott (1982: 1365). Their model results are consistent with U.S. macro time-series standard deviations. Their model calibration consists of searching a set of model variables (4 preference parameters, 9 technology parameters, and 3 shock variances) to explain covariances of cyclical components.

Endogenous business cycle (EBC) growth theory assumes that internal factors like research and worker training can increase growth rate. Exogenous shocks are not considered. Hallegatte (2008) presents a small system of ordinary differential equations (NEDym) to model the business cycle without growth. He presents results compared to the 2001 EU economy. The cyclical business cycle behavior in NEDym is driven by an investment – profit instability. Of the models investigated in the literature, NEDym is small complete business cycle model and its steady state results are compared with 2001 EU-15 observed values. NEDym advances the static model of Solow (1956) by adding time dependency. We determine that the NEDym model is a good starting point notwithstanding its nonphysical wage and price predictions, see below. Our model differs from the NEDym model in that we replace the physics driving the business cycle and several of its governing differential equations.

Leading, coincident, and lagging indicators are compiled by the Conference Board (2000) and are closely watched by the economics community. Yamarone (2004:52) reports that, from 1960 to 1991, leading indicators that suggest future trends generally peak 3 to 15 months before the coincident or business cycle peak. Lagging indicators, follow GDP from 9 months lead to 13 months lag. Also, leading indicators generally peak 1 to 8 months before the coincident or business cycle trough. Lagging indicators lag from 3 to 21 months after business cycle troughs. Thus, there is a wide variation in leads and lags referenced to the GDP or coincident indicator peaks and troughs.

Business cycle variability in terms of duration has been examined by Harloff and Eacott (2009) over 32 U.S. business cycles. They find that, while each business cycle is different, recession and expansion duration correlates with total business cycle duration. Of these 32, they find 26 business cycles are "normal" and 7 are "super-recessions". Normal recessions have recessions shorter than expansions and super-recessions have recessions longer.

3 Business cycle dynamics model, BuCDyn

The present work develops a manageable business cycle model without trends. We ignore: politics and external exogenous shocks, government spending, export and import flows, banks or bond interest, technology or productivity gains, and maximum labor supply changes. We assume two households of capitalist and consumers. The sum of capitalist money supply, F, and consumer money supply, M, is assumed constant.

This pure cycle approach makes it difficult to compare computed results directly with actual U.S. data due to increasing: price level, technology gains, and maximum labor supply increases. For example from 1951 to 2009, the U.S. productivity factor, A, increases from 0.025 to 0.16 and price level, p, increases by a factor of 9 over this period. These long-term trends are ignored in the BuCDyn model. The business cycle variables that normally have units of $ are divided by price level in the model such that their units are quantity or amount. An equation for price level, p is included to obtain units of dollars.

As indicated above, our math model is based partly on the NEDym model, with several important changes. We change several NEDym equations because it predicts unrealistic wage changes and price levels. We worked with a version of the NEDym model with no resolution of these issues. For example, NEDym predicted nominal wages for the 2001 EU are plus and minus 55% relative to the mean nominal wage over a business cycle with α_{inv} of 1.7 (Hallegatte 2008: 71). Price levels oscillate from 2 to 18 with α_{inv} of 2.5 (Hallegatte 2008: 69). And much higher nominal wage oscillations, from 0 to 250, are computed for α_{inv} = 10 (Hallegatte 2008: 72).

3.1 Wage-employment equations

We develop wage and employment governing equations first examining predator-prey concepts and finishing with final wage-employment equations. We incorporate a nonlinear Phillips type of equation to quantify wage and employment changes.

Lotka (1920) and Volterra (1926) both formulated two differential equations that, when solved, provide periodic oscillations for two competing “populations” y and x, see below. Because of their application to biological populations these equations (1) and (2) are sometimes known as predator-prey equations, see Table 1. The prey is x(t) and predator is y(t). Time is t, and a_1, b_1, a_2, b_2 are constants. The interaction, or xy term, is positive for the predator and negative for the prey differential equations. In the prey differential equation (2), if the predator, y, is zero the prey, x, will increase. Similarly, in the predator differential equation (1), if the prey, x, is zero the predator, y, will decrease.

The predator-prey model suggests a form of the wage and employment differentials equations for our business cycle model. We employ two coupled differential equations for employed labor ratio, λ, and total wage ratio to pY, i.e. W/(pY). These two equations are sufficient for a business cycle to cycle. Goodwin (1967) modeled the worker pay and employment level as a predator-prey system. Keen (1995: 615) employs a related set of equations. Wage change is generally positive when employment, λ, increases, and conversely wage change is generally negative when employment, λ, decreases. The relationship between wages and employment is a Phillips curve or equation, $P(\lambda)$. We modify Keen’s Phillips equation constants, $P(\lambda)$, see equation (3). This modification is to practically limit maximum λ to 1.0.

The following governing differential equations for wages and employment follow from equilibrium statements. When employment is in equilibrium, i.e. $P(\lambda) = 0$, there is no change in total wages, W. The conservation equation for total wage ratio is (4). When wages are in equilibrium, e.g. $W_0 = (W/pY)$ there is no change in employment. Here W_0 is a constant in the present model. The conservation equation for employment ratio, λ, is (5). Once λ is computed the employed labor supply, L, is known from (6).

Table 1 BuCDyn wage-employment equations

Predator: $d/dt\,(y) = -a_2 y + b_2 xy$	(1)
Prey: $d/dt\,(x) = a_1 x - b_1 xy$	(2)
$P(\lambda) = 1/[1-\lambda]\ \exp(-57.911+55.162\,\lambda) - 0.0478$	(3)
$d/dt\,(W/pY) = (W/pY)[P(\lambda))]\,/\tau_{wag}$	(4)
$d/dt\,(\lambda) = \lambda\,[(W_0 - (W/pY))/\nu\,]\,/\tau_{emp}$	(5)
$L = \lambda\, L_{max}$	(6)

The wage and labor differential equations (4) and (5) are fundamental to our business cyclical model. The coefficients on the right hand side of (4) and (5), and initial conditions govern the business cycle trajectory in W/pY vs. λ coordinates (see below). Our development of (4) and (5) is a progression from Goodwin’s (1967) and Keen’s (1995) wage and labor models. In the Goodwin and Keen models, capitalists invest all of their profits, i.e. $W_0 = 1$, and the two respective time constants are 1.0. Here we define the equilibrium wage divided by price level times Y (to get dollars); W_0, is assumed to be 0.60. Time constants for wage (employment), τ_{wag} (τ_{emp}), are chosen to be variable and either 1.0 or 1.5 years (1.5 or 1.0) depending if the right hand side of (5) is positive or negative respectively. With time constants greater than one, the magnitude of the right hand side terms of (4) and (5) decrease and the solution time to equilibrium increases. In the NEDym model both time constants are 2 years. In Keen τ_{wag} and τ_{emp}, are 1.0, and $\nu = K/Y$. Here we define $\nu = AK/Y$. The wage-labor orbit or trajectory quickly stabilizes as is discussed below.

3.2 Governing Equations

The process of computing business cycles with differential equations is called "computational business cycle dynamics" here. Ordinary differential equations for rates of change of goods inventory, G, price level, p, capitalist money stock, F, capital stock, K, and consumer money stock, M, are listed below. These follow Hallegatte (2008) where possible. In addition to these variables, Hallegatte employed a differential equation for producer investment ratio, Γ_{inv}, to control the business cycle amplitude and period. In the NEDym model, business cycles are due to an investment vs. profit instability.

To address NEDym's unrealistic wage and price behavior noted above, we replace its wage and employment equations with (4) and (5). In the process we replace the NEDym investment vs. profit instability business cycle driver with a natural oscillation between wage change and employment change. We also replace NEDym's differential equation for producer investment ratio, Γ_{inv}, with an algebraic equation, see below. And we replace NEDym's linear Phillips equation with a nonlinear equation. In summary we replace three of seven of NEDym's governing differential equations, replace the physics driving the business cycle, replace the Phillips equation, redefine the producer investment ratio Γ_{inv} and introduce variable wage and employment time constants. The BuCDyn equations are (3)-(24) in Tables 1 and 2.

In equilibrium, the rate of change of goods inventory, G, is zero, i.e. production, Y, equals demand, D. Thus, the conservation equation of G, or rate of change of G, is (7), see Table 2. Several identity algebraic equations are also needed. We follow NEDym where possible. Consider that demand, D, equals consumption, C, plus investment, I, see (8). The conservation equation of price level, p, is assumed to be negatively proportional to goods inventory, G, divided by demand, see (9) where the proportionality constant is $\alpha_p = 3.6E\text{-}03$. Production, Y, is assumed to be dependent on a constant technology coefficient, A, total labor, L_{max}, (in millions of workers) and capital stock, K (in trillions of \$), and is modeled by a Cobb-Douglas (1928) equation, see (10). For this zero growth economy, constant productivity, A, maximum labor, L_{max}, and sum of money supply are constant. In equilibrium, the consumer's income of wages plus dividends are consumed and/or saved. Thus, the rate of change of consumer money stock, M, is equal to total wages plus dividends minus price consumption minus savings, see (11). Total wages, W, are determined from the wage differential equation (4) solution. Capitalists use a portion of F, α_F, to pay for physical investments, pI, and dividend, Div, payout, see (12).

Our constant coefficients are consistent with the NEDym model except where noted. Dividends are computed from (13). We assume investment is dependent on investment ratio, Γ_{inv}, using rate of capitalist money stock, $\alpha_{F,}$ and capitalist money stock, F, see (14). Thus, dividend, Div, is dependent on investment ratio, Γ_{inv}, and producer liquid assets, $\alpha_{F,}$ assumed to be 0.2, and capitalist money stock, F, see (15). We define the investment coefficient Γ_{inv} to be proportional to normalized gross profits, ξ_p, see (16). We define normalized gross profit, ξ_p, to be gross profits, Π, per unit GDP in \$, pY, plus a constant of 0.2 to insure positive values, see (17). Our equation for Γ_{inv} is different where Γ_{inv} is determined in NEDym by solving an ordinary differential equation. Gross profits, Π, are total sales, p D, minus total labor costs, W, see (18). Consumer consumption, C, is proportional to (1 - savings ratio) times consumer stock of money, M, see (19). Where the savings rate, $\gamma_{save,}$ is assumed to be constant at 0.3 and the using rate of consumer money stock, α_M is assumed to be 0.2. Consumer savings, S, is assumed to be proportional to consumer money stock, M, see (20).

In equilibrium investment, I, equals capital stock, K, minus depreciation. Depreciation is assumed to be capital stock divided by the capital stock depreciation years. We assume a useful life of capital stock of 20 years and the time constant, $\tau_{dep,}$ is 20 years. The conservation of capital stock K, or rate of change of K, is investment, I, minus depreciated capital stock, K, is given by (21). At equilibrium, gross profits, Π, plus consumer savings, S, equal payouts of dividends, Div, and investment, pI. Thus, the rate of change of capitalist money stock, F is given by (22). Because the total money supply is assumed constant, the rate of change of consumer money stock, M, is equal and opposite of the rate of change of capitalist money stock, F, is given by (23).

By examining quarterly U.S. data from 2004 to 2010, we conclude that goods inventory, G, is usually quite close to zero. Employing G=0, we compute the using rate, $\alpha_{M,}$ of consumer money stock, M, endogenously as part of the numerical solution, see (24). For the U.S. 2007 solution presented below the using rate, α_M of consumer money stock, M, varies around 0.20. In Hallegatte (2008) α_M is 0.2.

Table 2 Other BuCDyn equations

$dG/dt = Y - D$	(7)
$D = C + I$	(8)
$dp/dt = - p\ \alpha_p\ G/D$	(9)
$Y = A\ L^{2/3}\ K^{1/3}$	(10)
$dM/dt = W + Div - (pC + S)$	(11)
$pI + Div = \alpha_F\ F$	(12)
$Div = \alpha_F\ F - pI$	(13)
$I = \Gamma_{inv}\ \alpha_F\ F/ p$	(14)
$Div = \alpha_F\ F - \Gamma_{inv}\ \alpha_F\ F = (1- \Gamma_{inv})\ \alpha_F\ F$	(15)
$\Gamma_{inv} = 0.5\ \xi_p$	(16)
$\xi_p = \Pi/(pY) + 0.2$	(17)
$\Pi = p\ D - W$	(18)
$C = (1- \gamma_{save})\ \alpha_M\ M/p$	(19)
$S = \gamma_{save}\ \alpha_M\ M$	(20)
$dK/dt = I - K/ \tau_{dep}$	(21)
$dF/dt = \Pi + S - Div - p\ I$	(22)
$dM/dt = - dF/dt$	(23)
$\alpha_M = pAL^{2/3} K^{1/3} /[(1- \gamma_{save})\ M] - \Gamma_{inv}\ \alpha_F\ F/[(1- \gamma_{save})\ M]$	(24)

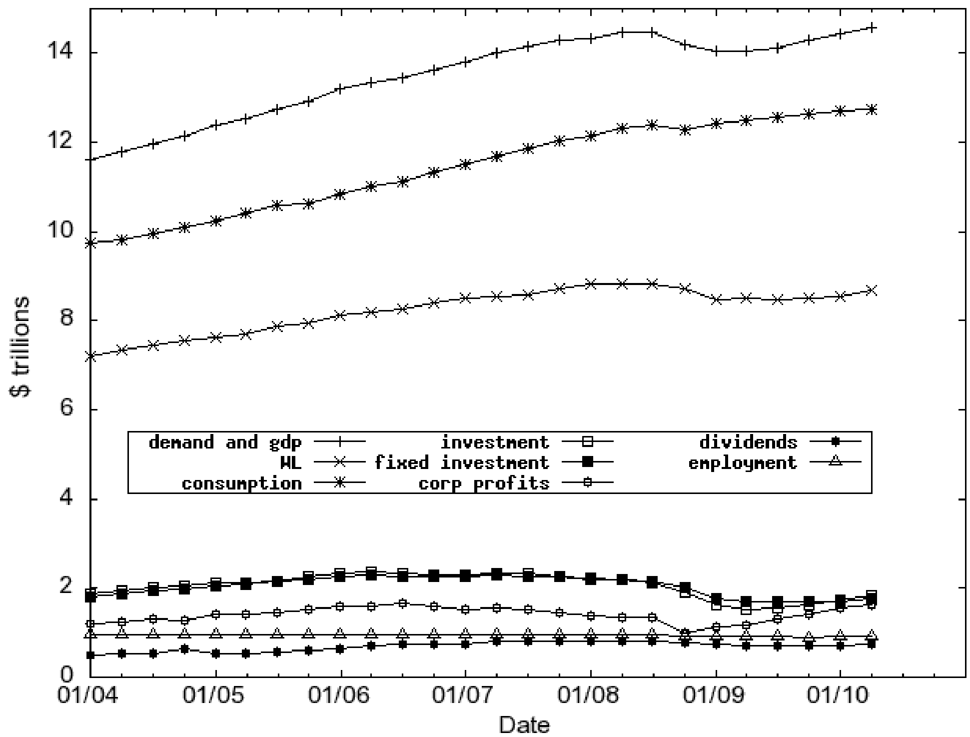
4 The U.S. economy, circa 2007

In this section, a general view of the U.S. economy is presented in terms of GDP distribution. National income accounting as is pertains to our two household model is discussed. Consumption of goods and services by domestic households account for about 2/3 of GDP, Y. Consumer consumption is the sum of: consumer consumption (durables, non-durable), government goods and services, and net exports. Government spending for goods and services accounts for about 1/5 of GDP, Y. Investment spending for new capital stock (fixed investment) plus inventory investment accounts for about 1/6 of GDP, Y. Investment is made up of both business and residential fixed investment, and inventory investment. The capital stock for the U.S. is typically 2.4 to 3.2 times Y from 1951 to 2010-II. Average capital stock for 2007-2008 from BEA (2010) is $44.56 trillion. Total wages are the sum of wages & salaries. National income is the sum of total wages and corporate profits. There are three different approaches to national income accounting that provide identical results. These are product, income, and expenditure. These approaches are the same because total production = total income = total expenditure. Gross domestic product = gross national product – net factor payments from abroad. In BuCDyn, there are no payments from abroad so GDP = GNP. The expenditure approach computes GDP, Y from consumption, C, investment, I, government purchases of goods and services, G, and net exports, NX, or exports - imports. In BuCDyn there are no G or NX so Y = C + I. In national accounting, real GDP, Y, is adjusted for price change to reflect only quantity changes.

Because there are no government spending or goods in our math model, the U.S. values from BEA, Table 2.1 (2010), are adjusted as if there is no government spending and these results are compared with the calculations. Therefore, the 2007 values, on a simplified accounting system with no government, are: (a) GDP = $13.84, consumption (no government) = consumption + government goods & services + exports + imports = 9.73 + 2.69 + 1.64 –2.35 = $11.72, (b) investment (no government) = investment + inventory = 2.13 + 0 =$ 2.13, (c) wages (no government) = wages and salary + 0.67 proprietors income + business current transfer payments + current surplus of government enterprises = 7.87 + 0.67 (1.63) + 0.09 – 0.01 = $9.04, (d) corporate profits (no government) = corporate profits + 0.33 proprietors income + rent + net interest + taxes on output and imports = 1.6 + .33 (1.63) + 0.07 + 0.96 = $3.77. Proprietor's income is partly attributed to wages and partly attributed to profits. Values are in trillions of dollars.

GDP, demand, consumption, total wages, investment, fixed investment, corporate profits, employment, and dividends for U. S. business data from 2004-II to 2010-II are illustrated in Fig. 1, respectively. Total wages/GDP vs. employment, quarterly data 1948 to 2010 are illustrated in Fig. 2. For the time between 2006-I to 2010-II, W_0 is approximated from Fig. 2 to be 0.60. It is a wage/pY symmetry value. This value appears to vary slightly from 0.60 to 0.63 for the last 62 years. In the total wage, we include 0.67 of proprietor's income. The total range in employment ratio is from 0.89 to 0.975 and wage ratio varies from 0.59 to 0.65 over these decades. Specific trajectory paths of wage vs. employment most likely relate to specific U.S. circumstances including labor productivity and labor strengths. Our calculation computes a single trajectory in the total wage vs. employment space as indicated below.

Fig. 1 U.S. business cycle data from 2004-II to 2010-II, GDP, demand, consumption, total wages, investment, fixed investment, corporate profits, employment, and dividends (data source: BEA, T 1.12, 2010)



The time series of Fig. 1 are segregated into leading, coincident, and lagging time series in Fig. 3-5. The variables are scaled between 0 and 1 by employing maximum and minimum values over a complete business cycle from troughs from 2001 to 2009. The normalized values are: y = (Y-Ymin)/(Ymin – Ymax). In this non-dimensional manner, business cycle data are easier to compare with our purely cyclical numerical calculation results below.

Phase lag designation is with respect to industrial production index. Industrial production index, IPI is used in place of GDP where IPI has units of amount. It is determined qualitatively around the maximum IPI of December 2007. From these non-dimensional time series, we conclude that the U.S. data leading variables are gross profits and investment. Coincident variables are IPI, demand, and employment. And lagging variables are: consumer consumption, dividends, and wages. The phase lead/lag, between leading, coincident, and lagging time series is not constant throughout the business cycle. As discussed in the section 2 lead and lag times are highly variable over different business cycles and lagging indicators sometimes become leading indicators.

Fig. 2 Total Wages/GDP vs. employment, quarterly data 1948 to 2010, total wages assume 0.67 of proprietor's income (data sources: U.S. Department of Labor, 2010, Table 1.12)

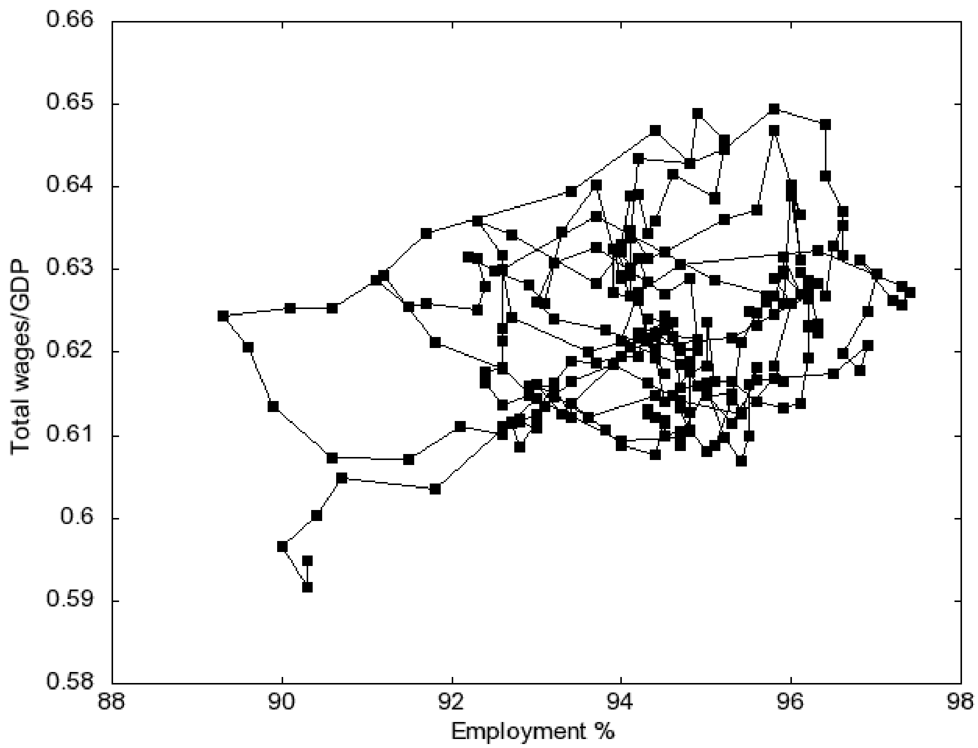


Fig. 3 Leading U.S. time series industrial production index, gross profits, investment and employment, normalized (Y-Ymin)/(Ymin – Ymax) within the last business cycle troughs in 2001 to 2009 (data source: BEA, 2010, T 1.12, Board of Governors of the Federal Reserve System, G17, IPI)

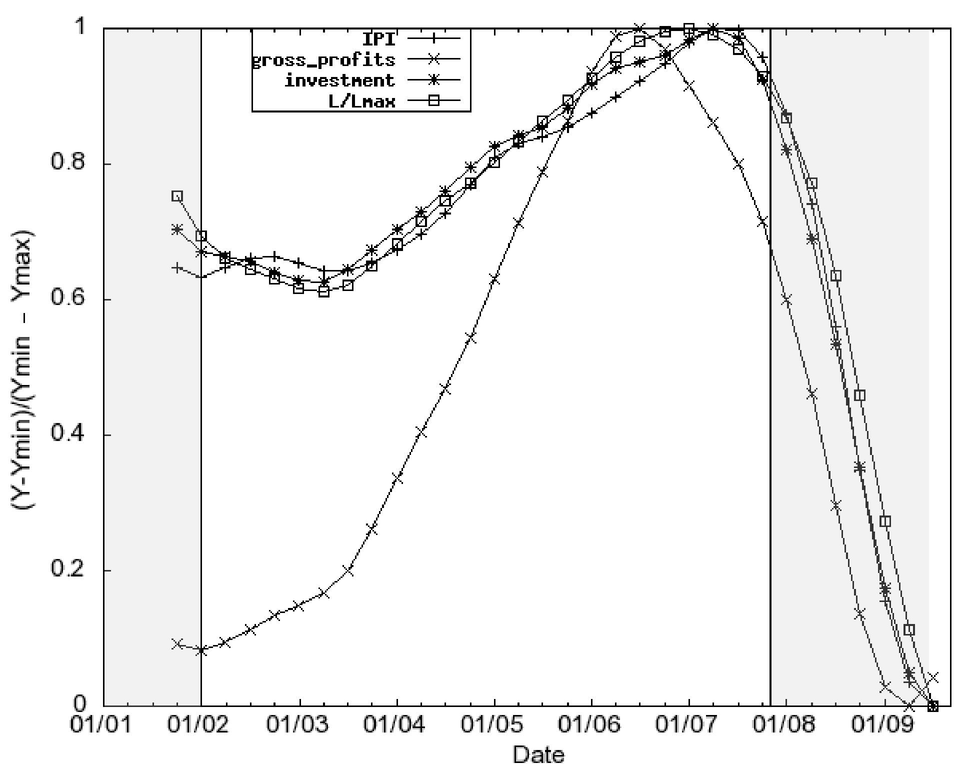


Fig. 4 Coincident U.S. time series industrial production index and wages, normalized (Y-Ymin)/(Ymin – Ymax) within the last business cycle troughs in 2001 to 2009 (data source: BEA, 2010, T 1.12, Board of Governors of the Federal Reserve System, G17, IPI)

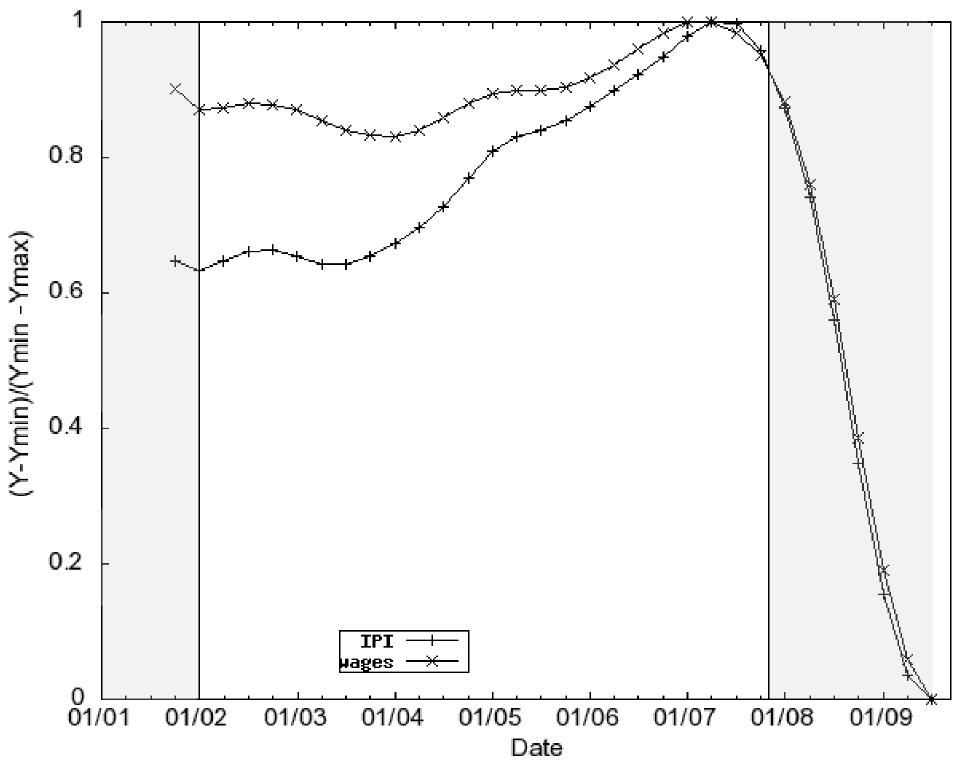
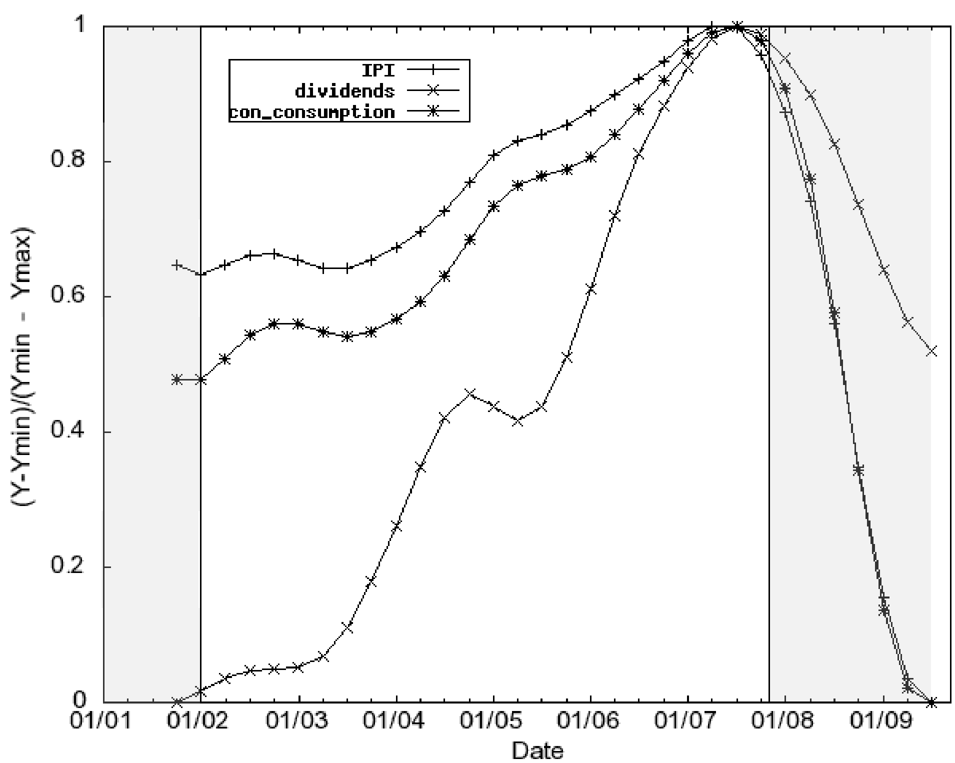


Fig. 5 Lagging U.S. time series industrial production index dividends, and consumer consumption, (Y-Ymin)/(Ymin – Ymax) within the last business cycle troughs in 2001 to 2009 (data source: BEA, 2010, T 1.12, Board of Governors of the Federal Reserve System, G17, IPI)



5 Prediction and data comparison

A picture is presented above of highly variable business cycle attributes. In contrast to this unsteady and changing process, we model and compute a specific steady state business cycle by solving the system of equations (3) – (24) with initial conditions until the time series are repeatable (see below). Initial conditions selected for the 2007 US business cycle example are: $F_0 = 53.61$, $G_0 = 0.0$, $K_0 = 60.66$, $L_0/Lmax_0 = 0.9$, $P_0 = 1.$, $W_0/Y_0 = 0.60$, $M_0 = 43.29$. The numerical integration procedure we employ is time accurate. In computing the 2007 U.S. economy with different money stock sums of capitalist and consumer money stock, F + M, we find that the final value of F is relatively insensitive to the initial sum of money stock. The initial value of M is judiciously selected to get a reasonable savings value.

A goal of the computer simulation is to obtain a cycling, steady state, non-trending business cycle. In order to obtain this, the computer program is run for a long time to "wash out" the unsteadiness to get purely cyclical results. This also means that capital stock, K no longer trends. Depending on initial conditions, this equilibrium time might take 80 or 100 years. These calculations typically take less than one minute on a desktop XP computer. Unsteady interactions within the business cycle system need a long time to settle out before achieving a steady state periodic oscillation. Thus, we conclude that actual business cycles are never in equilibrium and unsteadiness always present. This conclusion may explain why various business cycles, while qualitatively similar, are never the same. . This impedes the comparison between actual and computed steady state cyclical time series.

For the U.S. we find that inventory, G, is kept at zero by capitalists. Thus G is small and set to 0.0 in the math model. Therefore for the solution below we set dG/dt (7) to zero, dp/dt (9) = 0, and price level, p, is 1.0. Experimentation with nonzero G leads to reasonable price and wage values, with equation (9) employed. Thus, we think the price equation (9) is not responsible for the nonphysical large price level and wage level changes computed by Hallegatte (2008).

According to the NBER (2011), the last business cycle (or GDP) peak was December 2007 and trough was June 2009. Thus, the last recession was about 18 months. The computer output time is referenced such that the zero year corresponds to the end of December 2007, i.e. at the start of the last recession. Positive and negative years are relative to this date in Figs. 6-8. Average values over the last year in the computer model, from –1 to 0 years are listed in Table 3. The average 12-month observed values are listed both with and without government accounting. Averaged 12-month computed GDP is the same as the actual value of \$13.84 T. Computed average total wages of \$7.94 T compare with \$9.04 T with no government accounting and \$7.86T with government accounting. Computed average consumer consumption of \$10.66T compares with \$11.72T with no government accounting and \$9.81 T with government accounting. In general, the computed average 12-month values compare well with the no government accounting case, except gross profits and dividends are over predicted as shown in Table 3.

Fig. 6 illustrates how the computed GDP varies through out the computed business cycle. The variation with time is asymmetric. Computed recession is from 0 to 2 years where GDP drops. Computed expansion is from 2 to 5 years where GDP rises. Computed GDP is repetitive and the business cycle is in steady state.

Fig. 6 Computed GDP around 2007, period is 5 years, expansion 3 years before peak, and recession 2 years after peak

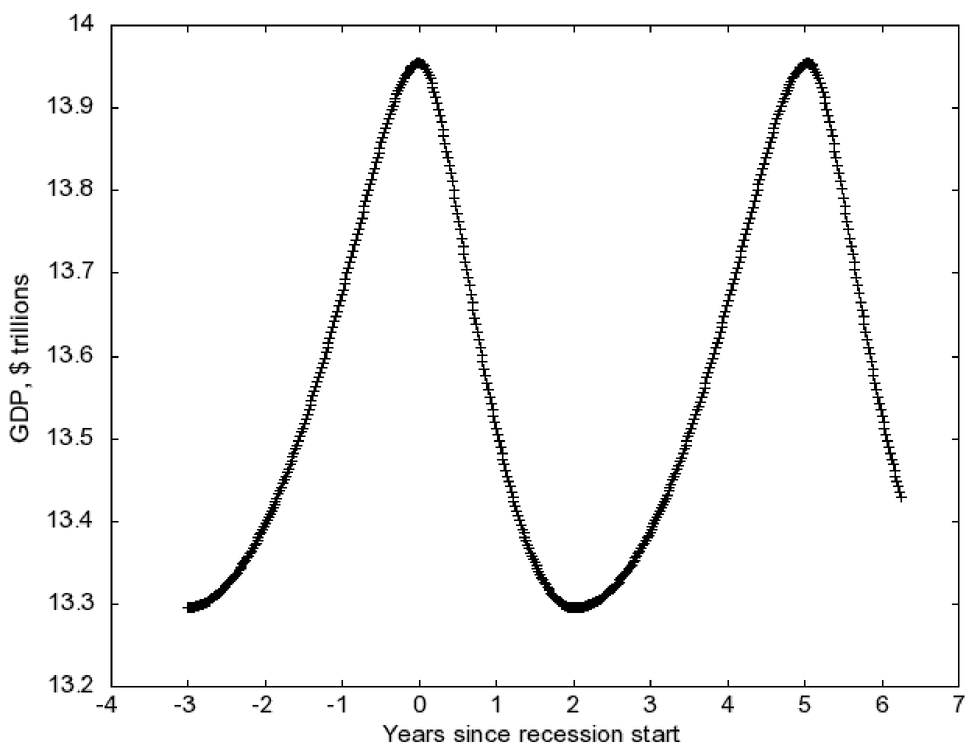


Table 3 U. S. 2007 business cycle, simplified with no government, and computed with BuCDyn

Business cycle variable symbol	Description	2007 computed, averaged over 12 mo. before peak	2007 observed values, with no government accounting	2007 observed[a] values, averaged over 4 quarters
Y = D	Production=demand	13.84	13.84	14.06
$\lambda = L/L_{max}$	Employed worker /153.121	0.953	0.95	0.95
W	Total annual wages	7.94	9.04	7.86
C	Consumer consumption	10.66	11.72	9.81
S	Consumer savings	2.80 ($S = \gamma_{save}\,\alpha_M\,M$)	2.13 (S = I)	2.66
Π	Gross profits	5.88	3.77	1.51
Div	Annual dividends	6.90	4.81 (Div = C + I -W)	0.79
K/τ_{dep}	Consumption of fixed capital, depreciation	3.04	1.69	n.a.
I	Physical investment	3.16	2.13	2.78
K[1]	Capital stock	60.85	n.a.	44.56
F[2]	Capitalist money	50.28	n.a.	n a
M[2]	Consumer money	46.62	n.a.	n a

Note: units are trillions of U.S. dollars except ratios that have no units. [a] Source: BEA (2010).

When in steady state capital stock, K, no longer changes and Investment, I, $= K/\tau_{dep}$. Thus Investment, I, = 60.85/20 = 3.04. Savings, S, $= \gamma_{save}\,\alpha_M\,M$ = 2.80, see Table 2. Capital stock, K, is determined by the model solution. For U.S. data, which includes a government household, from 1951 to 2009 K/Y varies from 2.4 to 3.2. Our computed average over a business cycle K/Y = 4.4. Thus, our computed steady state capital stock is higher than actual data. It is possible that the actual U.S. capital stock, K, reported as 44.56 is not a steady state value.

The 2007 U.S. total income with no government (W + Div or 9.04 + 4.81) is \$13.85 T, see Table 3. This is the same as GDP, Y \$13.84 T. Of the available money (S + Π or 2.13 + 3.77) \$5.90 T, part is distributed to capital investment, I, and the rest to dividends (I + Div) for a total of \$6.94 T. This is more than the available money \$5.90 T by about \$1.04 T.

Final conditions at t_{max} of 120 years are: F=50.13, G=0.0, K = 60.72, L = 147.2 M, P = 1.0, W/Y = 0.62, M = 46.77. Conditions averaged over the last business cycles are: F =50.28, G =0.0, K = 60.85, L/Lmax = 0.953, p = 1.0, W = 7.94, M = 46.62, GDP, Y = 13.84. These are in units of \$T, except price level in \$, and dimensionless ratios.

Cyclical steady state computations, after initial transients have died out, are illustrated in Fig. 7. These values compare with Fig. 1.

As illustrated in Fig. 3-5, actual business cycle data from October 2001 to July 2009 indicate that leading series are investment, gross profits and employment. Coincident series are GDP and wages. Lagging series are dividends and consumer consumption. As illustrated in Fig. 8, non-dimensional scaled 0-1 results indicate that the leading series are investment and gross profit. Coincident series are GDP, employment, and consumer consumption. Lagging series are dividends and wages. Thus there is agreement on the phase between actual and calculation for leading series of investment and gross profit. GDP is a coincident series by definition. Also there is agreement for lagging series of dividends. There is not phase agreement for employment, wages, or consumer consumption. This lack of agreement might be associated with the actual business cycle not being in a steady state periodic cycle as previously mentioned.

Computed wage share vs. employment is presented in Fig. 9. As illustrated, actual data from 2006-I to 2010-II is similar to the upper portion of the trajectory. A closed counter-clockwise processing trajectory of wage vs. employment is indicated. The overall computed magnitude of both the wage share and employment variation encompasses most of the actual data in Fig. 3. We think the current simulation is representative of the present business cycle where the employment variation is from 0.89 to 0.97.

Fig. 7 Computed GDP around 2007, period is 5 years, expansion 3 years before peak, and recession 2 years after peak, gross domestic product, total wages, dividend, gross profits, and investment

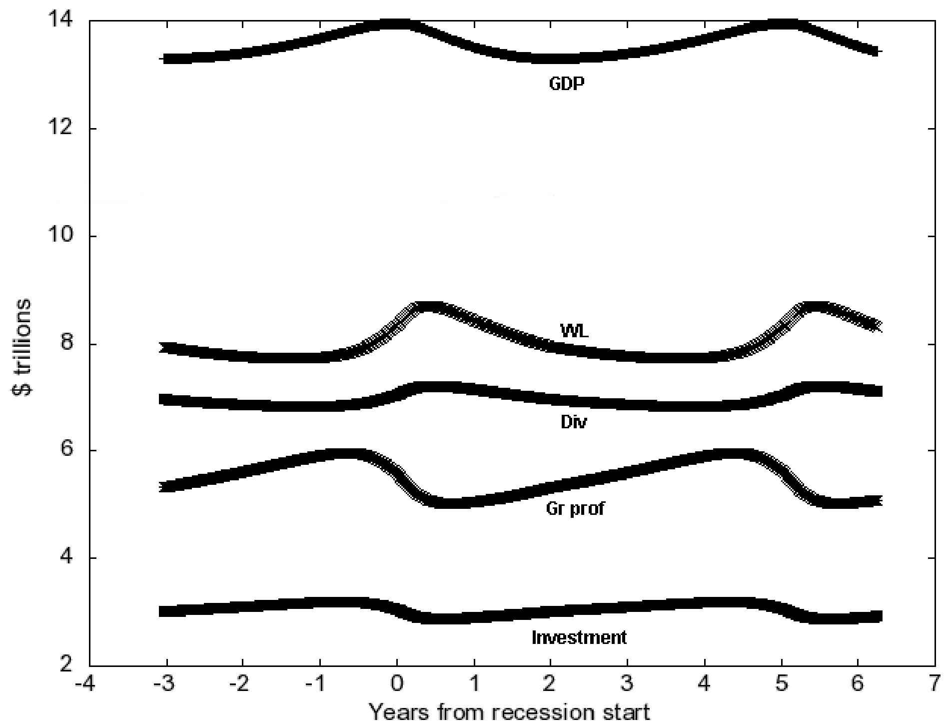


Fig. 8 Computed non-dimensional business cycle vs. years from start of recession, leading series are investment and gross profits, coincident series are consumer consumption, employment, and Gross Domestic Product, lagging series are dividends and total wages

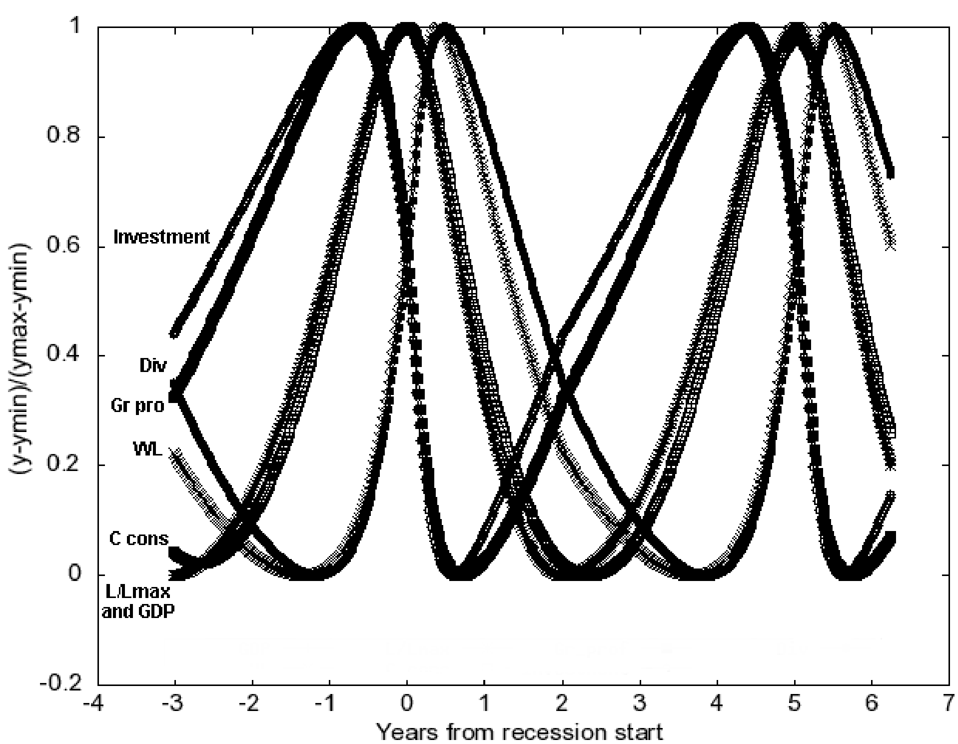
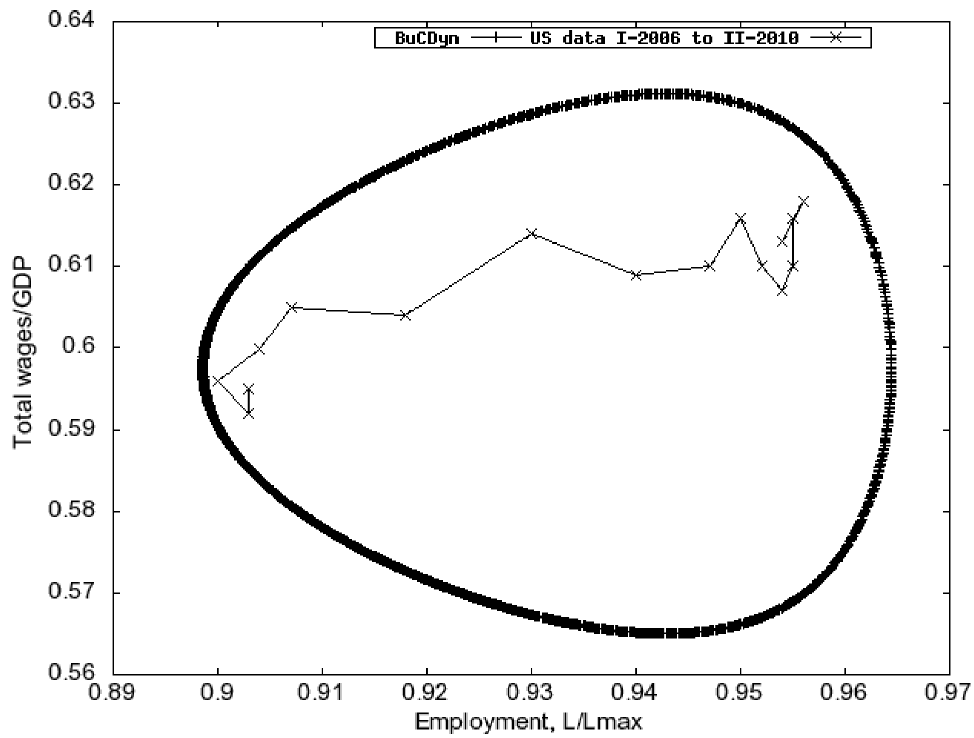
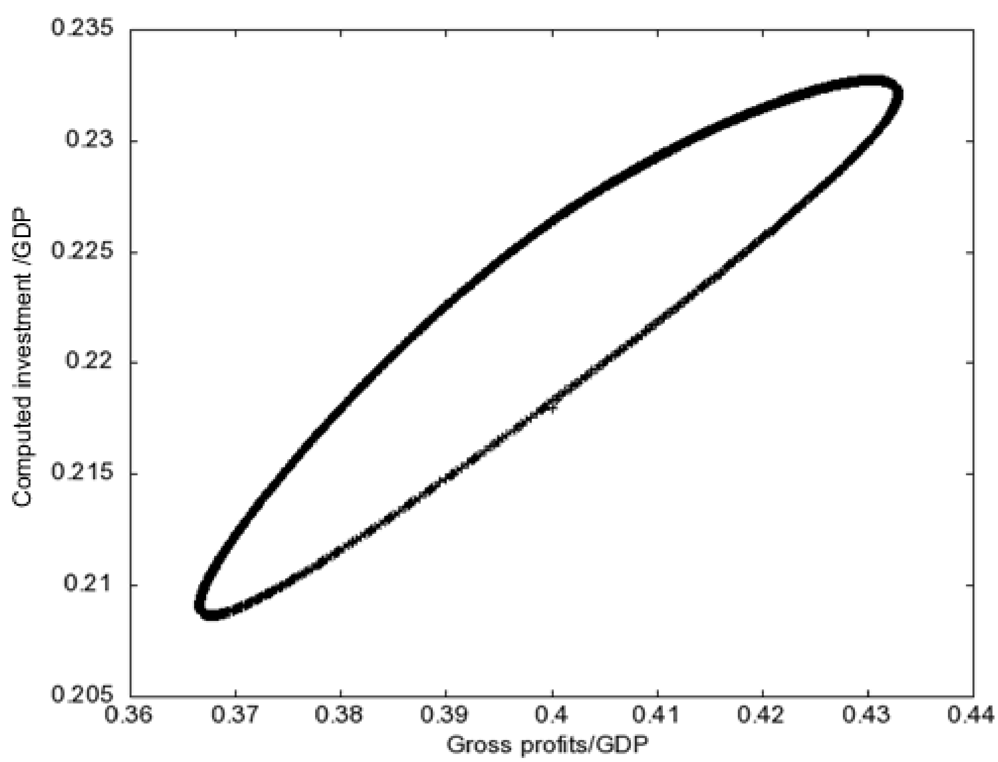


Fig. 9 Computed total wages/GDP vs. employment, U.S. data from I-2006 to II-2010, procession is counterclockwise (data source: U.S. Department of Labor, unrate, and BEA table 1.12)



The computed investment/GDP ratio vs. gross profits/GDP is illustrated in Fig. 10 to cycle in a nonlinear fashion and is double valued. This behavior is reasonable given that the business cycle is repetitive when in steady state. The procession is clockwise. Although not illustrated, the computed investment ratio, Γ_{inv}, varies non-sinusoidally from 0.285 to 0.318 over the business cycle.

Fig. 10 Computed investment/GDP vs. gross profits/GDP, procession is clockwise



Summary and conclusions

Computed business cycle values are compared with observed U. S. 2007 values. The computed business cycle is asymmetric with a period of 5 years, expansion of 3 years and recession of 2 years. In general, the computed average 12-month values compare well with the no government accounting case. These include: GDP, total wages, consumer consumption, consumer savings, and physical investment. These results are encouraging. However, gross profits and dividends are over-predicted by the model. We compute the business cycle until the cycle becomes repetitive and call this steady state. When steady state is reached, computed capital stock no longer changes. Time to reach this steady state is 80 to 100 years depending on initial conditions. In spite of the large number of years computed, actual computer solution time is less than one minute on a desktop computer. The numerical integration procedure is time accurate and the model could be employed to analyze the interactions throughout the equilibration process. Because of this long time needed to reach steady state, we conclude that actual business cycles are never in equilibrium and unsteadiness is always present. This feature may have profound influence over how "unsteady" business cycles are examined in the future. Further, it is obviously more difficult for a central bank to control an unsteady process than to control a steady state process.

The physics that controls and drives the BuCDyn business cycle is an interaction between labor change and wage change. Economies with different employment vs. wage characteristics, due to different work ethics or other issues, would have different business cycle characteristics. These characteristics may be dynamic. The computed wage-labor orbit/trajectory is quite stable and quickly becomes repetitive. This is in contrast to the long time required for other economic time-series to steady out. The phase lead/lag, between leading, coincident, and lagging time series is not constant throughout the business cycle.

Actual business cycle data from October 2001 to July 2009 indicate that leading series are investment, gross profits and employment. Coincident series are GDP and wages. Lagging series are dividends and consumer consumption. Calculations indicate that the leading series are investment and gross profit. Coincident series are GDP, employment, and consumer consumption. Lagging series are dividends and wages. Thus, there is agreement on the phase between actual and calculation for leading series of investment and gross profit. Also, there is agreement for lagging series of dividends. There is not phase agreement for employment, wages, or consumer consumption. This phase disagreement may simply be another indication of unsteadiness of actual business cycle data. Thus, with interactions always present it is difficult to understand, evaluate, control, and predict actual business cycles. We think the BuCDyn model is useful to researchers and consultants to government and business. An example of use of the model would be to examine various amounts of money stock and capital stock effects on employment, wages, and production. Another use could be to examine the interaction terms, effects, and processes before the model is in steady state. The simplicity of the model is such that it can be further expanded to incorporate complexities such as changing technology and labor supply, incorporate a government household, and include bank interest.

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