

U.S. Business Cycle Math Quantification

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Gary J. Harloff, PhD, e-mail: Harloff@covad.net
Eve E. Eacott, BSEE, e-mail: EEacott@covad.net

Harloff Capital Management, Harloff Inc.
795 Sharon Drive, Suite 226, Westlake, Oh 44145
440-871-7278

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Abstract U.S. Business cycle expansion and recession data from 1854 to 2001 are examined to develop new empirical equations for recession and expansion portions of business cycle duration. Three periods are analyzed to evaluate if the business cycle character changes over time. Two different types of recessions are found, normal and super-recession. Of the 32 complete cycles analyzed, 7 recessions are characterized as super-recessions. For 6 of the 7 super-recessions, the recession duration exceeds the expansion duration. The developed normal and super-recession business cycle models represent both recession and expansion data reasonably well.

Keywords business cycle, duration, empirical, expansion, recession, super-recession

1 Introduction

Policy makers, business and congressional leaders, business owners, managers, and economists currently depend on the NBER committee to announce business expansions and recessions dates. Unfortunately, the committee's pronouncements usually lag reality from 6 to 18 months. It is common sense that better policy and business decisions are made with realistic assessment of the business cycle situation. That is, shorter pronouncement time lags are better than long time lags. A current example that illustrates the importance of a short lag pronouncement is that several U.S. economic stimulus packages are being promulgated to ameliorate the current recession. Many policy makers worry if the stimulus is too late to avoid a deep recession and do not know if we are in a regular recession or something worse. At the business level, timely awareness of the expansion/recession economic landscape is obviously helpful in making strategic business decisions. In spite of the importance in timely quantifying expansion and recession dates, the NBER's lagged approach is the standard. Thus, there is a need to advance the quantification and analysis of U.S. business cycles. This paper is offered to benefit the economics and finance community including policy makers, business and congressional leaders, business owners, managers, and economists by advancing the quantification of U.S. business cycles.

Business cycles are quite different in that durations from 28 to 128 months are reported over 32 complete cycles from 1854 to 2001. The components are also diverse with recessions from 6 to 65 months and expansions from 10 to 120 months. Business cycle data are herein normalized to compare the recession and expansion components on a consistent basis. The authors are not aware of similar published work.

The paper is organized to: present business cycle expansion and recession data in section 2, analyze the business cycle data in section 3, introduce super-recession business cycles in section 4, analyze business cycle consistency in three time periods in section 5, evaluate recession duration in section 6, and present conclusions in section 7. The contributions of the paper are to quantify business cycle expansion and recession duration with empirical equations and to identify super-recession business cycles.

2 Data

Data from the Business Cycle Dating Committee of the National Bureau of Economic Research, NBER, are employed in this analysis. This committee defines dates of troughs and peaks in U.S. economic activity as well as expansion and recession duration times. A complete business cycle is defined to be the time between trough to trough. The expansion duration is defined to be the time between the trough and peak, and the recession duration is defined to be the time between the peak and the trough, see equation (1) below and Fig. 1. The data is organized such that a trough precedes a peak, i.e. an expansion proceeds a recession, see Table 1. The recession and expansion durations are normalized by the business cycle duration in order to analyze the various business cycles on a common basis. It is convenient to define expansion ratio (ER) to be expansion duration/business cycle duration (T), and recession ratio (RR) to be recession duration/ business cycle duration. The recession and expansion ratios are related to each other by the definition given in equation (2) below.

$$E + R = T \tag{1}$$

$$ER + RR = 1 \tag{2}$$

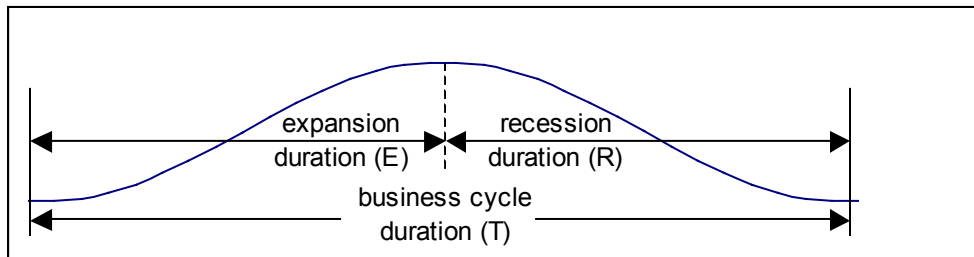


Fig. 1 Schematic of Business Cycle, expansion and recession durations

Table 1 NBER dates and durations of 32 complete expansion and recession from 1854 to 2001 (source: Business Cycle Expansions and Contractions www.nber.org/cycles.html accessed 2/5/2009)

Business cycle no.	Trough	Peak	Trough	Expansion (mo)	Recession (mo)	Total (mo)	Expansion /total = ER	Recession /total = RR
1854-1919								
1	Dec-1854	Jun-1857	Dec-1858	30	18	48	0.63	0.38
2	Dec-1858	Oct-1860	Jun-1861	22	8	30	0.73	0.27
3*	Jun-1861	Apr-1865	Dec-1867	46	32	78	0.59	0.41
4	Dec-1867	Jun-1869	Dec-1870	18	18	36	0.50	0.50
5*	Dec-1870	Oct-1873	Mar-1879	34	65	99	0.34	0.66
6*	Mar-1879	Mar-1882	May-1885	36	38	74	0.49	0.51
7	May-1885	Mar-1887	Apr-1888	22	13	35	0.63	0.37
8	Apr-1888	Jul-1890	May-1891	27	10	37	0.73	0.27
9	May-1891	Jan-1893	Jun-1894	20	17	37	0.54	0.46
10	Jun-1894	Dec-1895	Jun-1897	18	18	36	0.50	0.50
11	Jun-1897	Jun-1899	Dec-1900	24	18	42	0.57	0.43
12*	Dec-1900	Sep-1902	Aug-1904	21	23	44	0.48	0.52
13	Aug-1904	May-1907	Jun-1908	33	13	46	0.72	0.28
14*	Jun-1908	Jan-1910	Jan-1912	19	24	43	0.44	0.56
15*	Jan-1912	Jan-1913	Dec-1914	12	23	35	0.34	0.66
16	Dec-1914	Aug-1918	Mar-1919	44	7	51	0.86	0.14
1919-1945								
17	Mar-1919	Jan-1920	Jul-1921	10	18	28	0.36	0.64
18	Jul-1921	Mar-1923	Jul-1924	22	14	36	0.61	0.39
19	Jul-1924	Oct-1926	Nov-1927	27	13	40	0.68	0.33
20*	Nov-1927	Aug-1929	Mar-1933	21	43	64	0.33	0.67
21	Mar-1933	May-1937	Jun-1938	50	13	63	0.79	0.21
22	Jun-1938	Feb-1945	Oct-1945	80	8	88	0.91	0.09
1945-2001								
23	Oct-1945	Nov-1948	Oct-1949	37	11	48	0.77	0.23
24	Oct-1949	Jul-1953	May-1954	45	10	55	0.82	0.18
25	May-1954	Aug-1957	Apr-1958	39	8	47	0.83	0.17
26	Apr-1958	Apr-1960	Feb-1961	24	10	34	0.71	0.29
27	Feb-1961	Dec-1969	Nov-1970	106	11	117	0.91	0.09
28	Nov-1970	Nov-1973	Mar-1975	36	16	52	0.69	0.31
29	Mar-1975	Jan-1980	Jul-1980	58	6	64	0.91	0.09
30	Jul-1980	Jul-1981	Nov-1982	12	16	28	0.43	0.57
31	Nov-1982	Jul-1990	Mar-1991	92	8	100	0.92	0.08
32	Mar-1991	Mar-2001	Nov-2001	120	8	128	0.94	0.06
33	Nov-2001	Dec-2007	NA	73	NA	73	NA	NA

* **7 Super-recessions**: No. 3, 5, 6, 12, 14, 15, 20 are in bold. The sum of ER and RR values in Table 1 may not add to 1 due to rounding.

The NBER committee's definition of each business cycle peak and trough dates are announced with a lag between 6 to 18 months. For example, the last expansion phase ended

December 2007 and was announced 12 months later. Shorter lags are desirable for timely policy decisions. Various monthly series are analyzed to determine business cycle dates. These series include real GDP, from a private company, real personal income less transfer payments, and payroll employment. The committee has no fixed rule on weighing the different indicators in dating peaks or troughs, see Hall et. al.

3 Data Analysis

The RR vs. T data for the 32 business cycles are illustrated in Figure 2. An exponential equation is employed to fit the data, i.e.

$$RR = 0.696 \exp(-0.0157 T); R^2 = 0.359, \text{sigma} = 0.179 \quad (3)$$

The authors find the exponential representation adequate for this analysis; other math functional forms may be used. The RR equation (3) correlation coefficient is a modest 0.36 and explains 36% of the variation of the data. The standard deviation, sigma, is 0.179. This relationship indicates that RR decreases exponentially as business cycle duration increases. From Fig. 2 and errors associated with the model equation, several outliers are evident. Outlier determination is important to conclude if there is another “population”. Once the outliers are identified, then one seeks to determine if there is a physical reason for the outliers not fitting the model equations. In this paper we find two populations, one for normal and a different population called super-recessions. Of the 7 outliers noted in Table 1, 6 have a recession ratio greater than 0.5 and thus are different from the other normal business cycles. These recessions last longer than their corresponding expansions. The ability to test and discern two populations is important to adequately represent the business cycle data. This math model selection and outlier discernment process is not arbitrary and is part of the model building process often practiced in applied Engineering and Science communities. This model building process may not be well known in the economics and finance field.

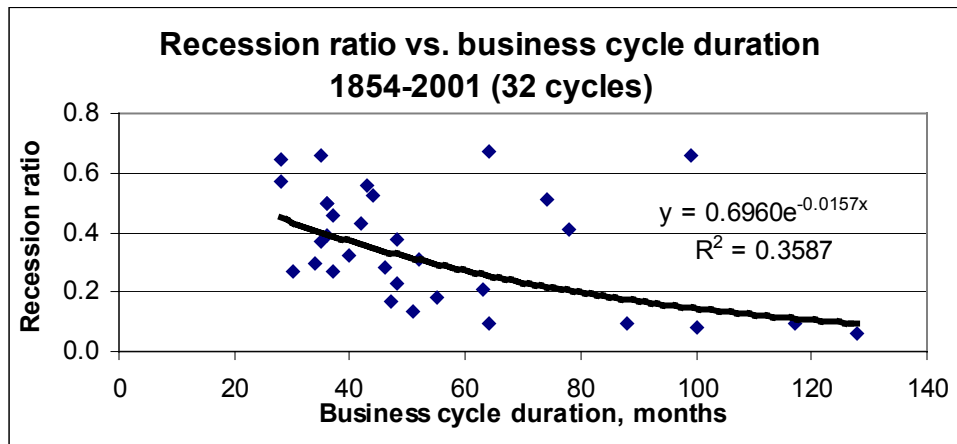


Fig. 2 Recession ratio vs. business cycle duration from 1854 to 2001, 32 complete cycles

4 Super-recessions

The procedure employed to further develop an equation for RR as a function of business cycle duration, is to drop out apparent outliers, one at a time, from the statistical data fit to see if the coefficient, exponent, and R^2 consistently improve. In this successive dropping out process, 7 business cycles are determined to be outliers that belong to a different data population. Six of the seven have recession ratios greater than 50% and all have larger equation errors than the normal data points. The 7 outliers are called super-recessions and are listed below in Table 2. As previously mentioned, this data analysis and model building process is not arbitrary and has been employed in the Aerospace industry for many years, see for example Harloff and Berkowitz and Harloff and Berkowitz. Separately subtracting these seven business cycle “outliers” from the population of business cycles changes the correlation equation and correlation coefficient appreciably. For example, the correlation coefficient improves from 0.359 to 0.748. Similarly, the standard deviation decreases from 0.173 to 0.097 when the seven outliers are dropped out of the correlation. The super-recession ratios are generally higher than 0.5 with the exception of the June 1861 to December 1867 business cycle with a 0.41 recession ratio, see Table 1.

Table 2 Super-recessions of seven business cycles. These super-recession business cycles are identified as different from normal recession business cycles. $RR = a \exp (b T)$

Trough	Peak (p)	Trough (t)	Expansion, t-p, months	Recession, p-t, months	Coefficient, a	Exponent, b	Standard deviation	Correlation coefficient, R^2
All data included					0.696	-0.0157	0.173	0.359
1854-1919								
Jun-1861	Apr-1865	Dec-1867	46	32	0.710	-0.0165	0.160	0.389
Dec-1870	Oct-1873	Mar-1879	34	65	0.826	-0.0204	0.157	0.559
Mar-1879	Mar-1882	May-1885	36	38	0.850	-0.0216	0.156	0.629
Dec-1900	Sep-1902	Aug-1904	21	23	0.827	-0.0214	0.156	0.634
Jun-1908	Jan-1910	Jan-1912	19	24	0.799	-0.0211	0.147	0.641
Jan-1912	Jan-1913	Dec-1914	12	23	0.759	-0.0205	0.107	0.642
1919-1945								
Nov-1927	Aug-1929	Mar-1933	21	43	0.753	-0.0213	0.097	0.748

With the 7 super-recessions removed, an improved equation for normal recessions is:

$$RR = 0.753 \exp (-0.0213 T) \quad (4)$$

The inverse of the duration T coefficient, 0.0213, is 46.9 months or 3.912 years and represents a characteristic time of the U.S. business cycle. This is close to a 4-year presidential cycle number. For comparison, the average normal business cycle duration is 53 months. This lends support for our analysis.

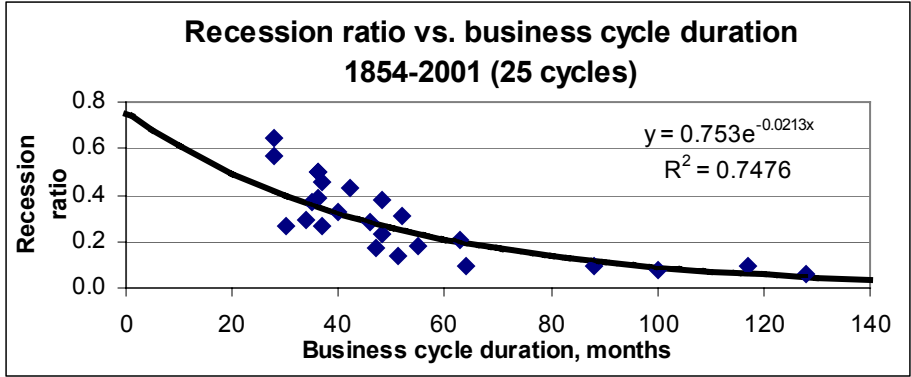


Fig. 3 Recession ratio vs. business cycle time with super-recessions removed, 1854-2001 (25 cycles)

Recession ratio is plotted vs. business cycle duration in Fig. 4 for the 7 super-recessions identified. The equation used for the constant line to compare with the super-recession ratio data in Fig. 4 is:

$$RR = 0.565, \text{ sigma} = 0.1240 \tag{5}$$

For this model, the recession duration for a super-recession increases in constant proportion to total business cycle duration.

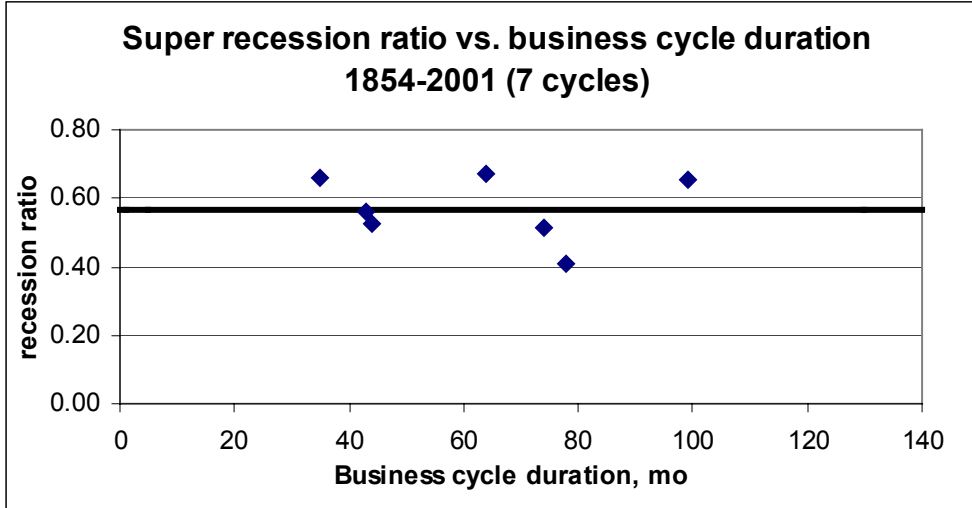


Fig. 4 Super-recession ratio vs. business cycle time, 1854-2001 (7 cycles)

5 Business cycle consistency from 1854-1919, 1919-1945 and 1945-2001

Zhou and Rigdon recently analyzed US business cycle duration data with the modulated power law process and concluded that a structural change occurred post WWII. They show average expansions (recessions) increasing (decreasing) from 1854-1919 to 1919-1945 to 1945-2001. We also examine the same three periods. Their analysis, in contrast to this paper, did not seek to

identify outliers and typical statistical analysis would miss two populations (normal and super-recessions). Thus our conclusions differ from their conclusions.

In Figures 5 thru 7 the normal RR data are plotted vs. business cycle duration for the periods 1945-2001, 1919-1945, 1854-1919 respectively. The line on these three figures is determined by equation (4). It is apparent that one equation reasonably fits all three periods reasonable well. We conclude that the business cycle character does not change over the three periods examined. Recall we have removed the 7 super-recessions from Figures 5, 6, and 7.

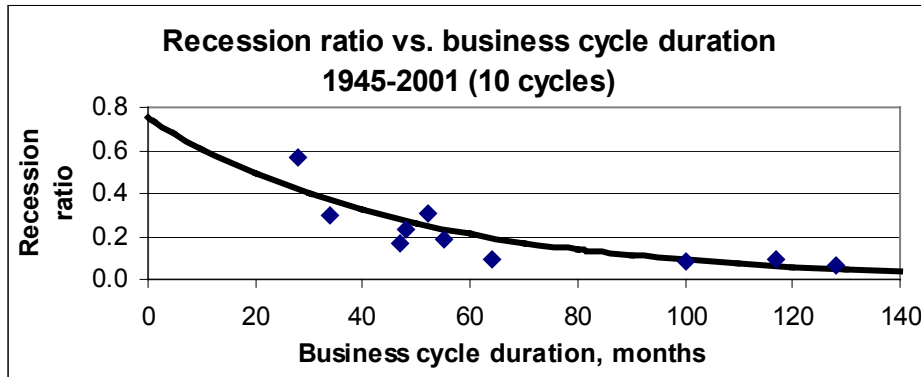


Fig. 5 Recession ratio vs. business cycle time, 1945-2001 (10 cycles)

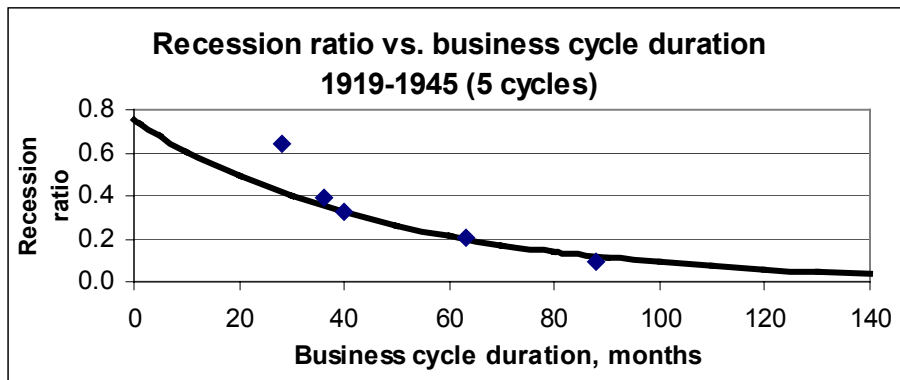


Fig. 6 Recession ratio vs. business cycle time, 1919- 1945 (5 cycles)

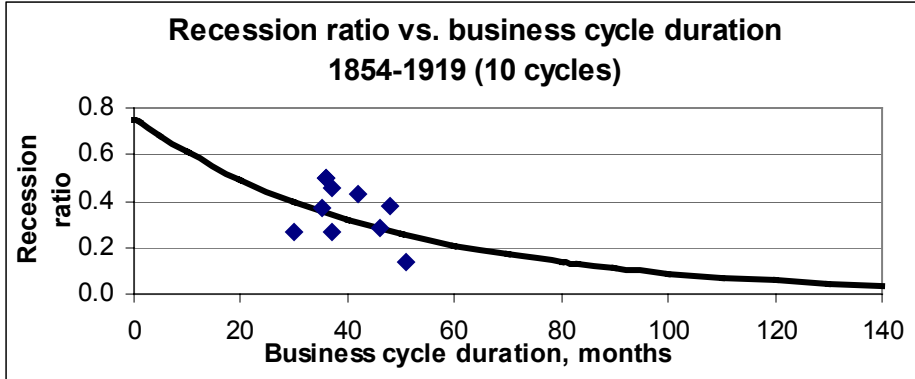


Fig. 7 Recession ratio vs. business cycle time, 1854-1919 (10 cycles)

6 Recession duration

Computed recession duration vs. actual business cycle duration data is illustrated in Fig. 8 for 25 normal recession business cycles. Error bars of plus and minus one standard deviation are illustrated in Fig. 8. Employing equation (4), the recession duration is linear with business cycle duration up to about 20 months, peaks around 13 months for a business cycle of around 45 months, and decreases at higher business cycle durations. The data is reasonable represented by the nonlinear prediction line.

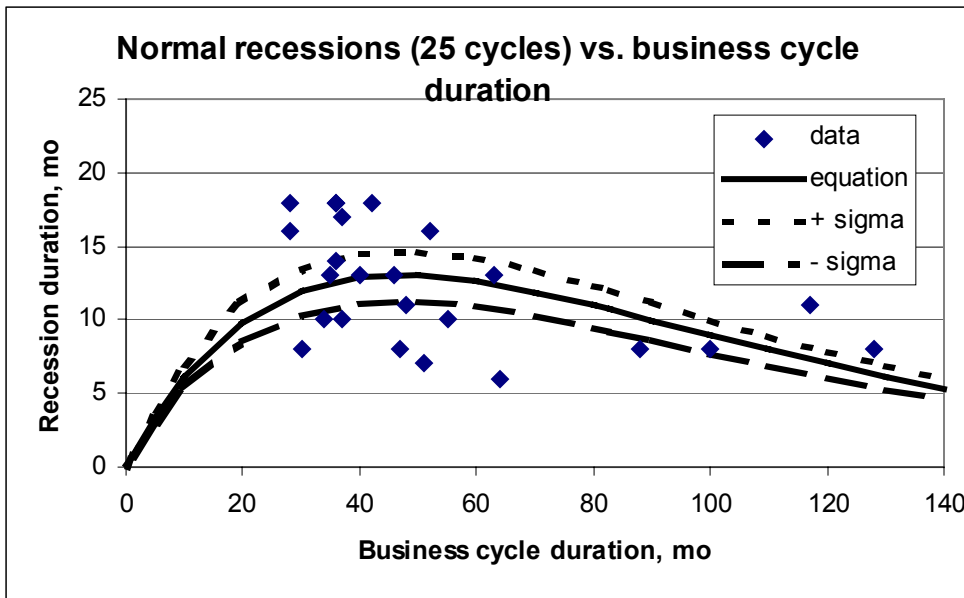


Fig. 8 Normal recession (25 cycles) vs. business cycle time, 1854-2001. Equation is $RR = 0.753 \exp(-0.0213 T)$, correlation coefficient = 0.748, $\sigma = 0.097$

The duration of super-recession business cycle data vs. business cycle duration is illustrated in Fig. 9. Equation (5) is employed. Recession duration is a constant 0.565 times the business cycle duration. Data from the 7 super-recessions are plotted in the figure and one

standard deviation error bands are shown. The data are well represented by the linear prediction line.

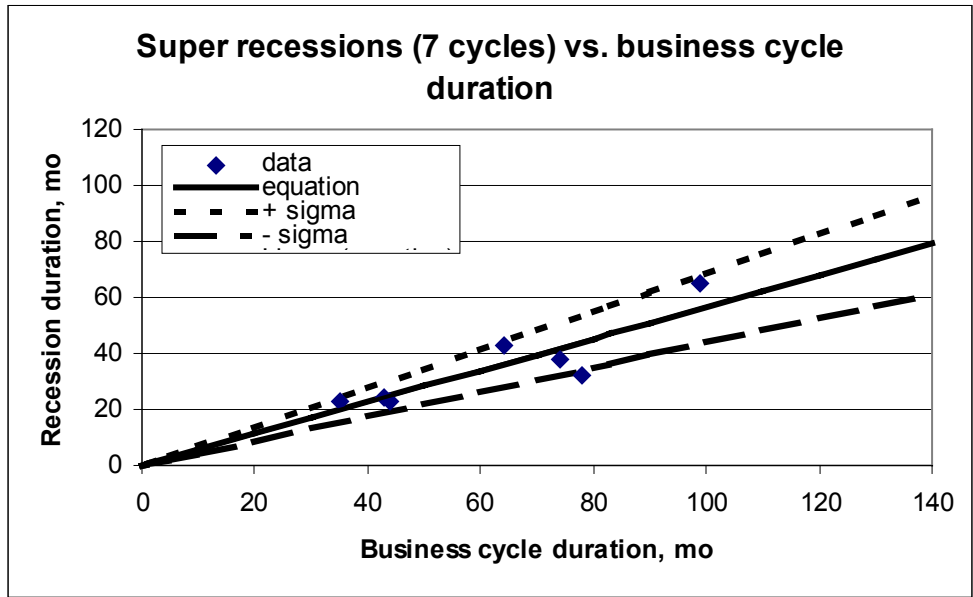


Fig. 9 Super-recession (6 cycles) vs. business cycle time, 1854- 2001.
 $RR = 0.565$, $\sigma = 0.12401$

The expansion duration for both 25 normal and 7 super-recessionary business cycles are illustrated in Fig. 10. These data is also well represented by the prediction lines.

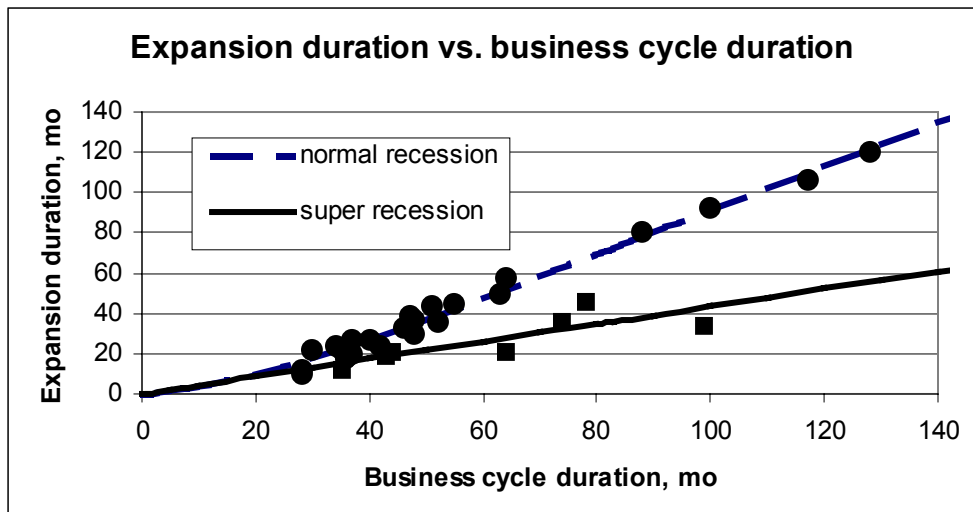


Fig. 10 Expansion time for normal and super-recession business cycles vs. business cycle length, 1854-2001

7 Conclusions and implications

We find two types of recessions in US business cycle data from 1845 to 2001. These two types are (1) normal recessions and (2) super-recessions. Characterization of the two recession types is important. Policy makers who seek to ameliorate recessions need to know if they are normal or super. Our analysis enables recession duration to be predicted, possibly months before the NBER pronouncement. This early indication of the recession duration should be important to policy makers, business and congressional leaders, business owners and managers, economists and others.

Normal (super-) recessions are generally characterized with recession duration less (greater) than expansion duration. Normal recession ratio depends nonlinearly on business cycle duration and decreases exponentially with increasing business cycle duration above 45 months. Super-recession duration is linear with business cycle duration. Our normal and super-recession business cycle models represent both recession and expansion data reasonably well. We find little difference in the business cycle quantitative character for 1854-1919, 1919-1945, and 1945-2001 periods. Six of the super-recessions are found in the 1854-1919 period. The seventh super-recession, the great depression, is found in the 1919-1945 period.

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